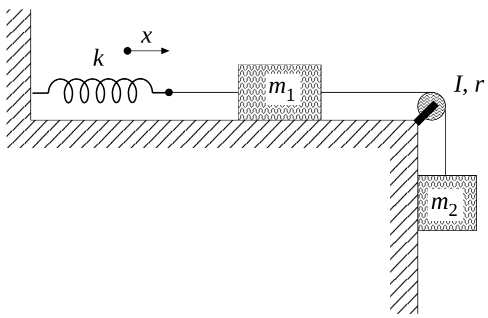
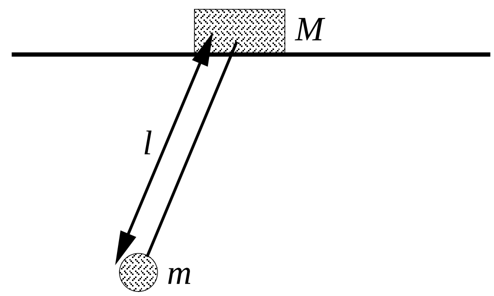
1. Masses m_1 and m_2 are connected in the frictionless spring-and-pulley system shown below. The spring has spring constant k; the pulley has radius r and moment of inertia I. The configuration of the system is described by x; x is zero when the spring is relaxed. Find the Lagrangian for this system. Find the Hamiltonian for this system.





2. Consider the motion of a frictionless glider (mass M) moving on a level air track with a pendulum (bob mass m, length l) attached to the glider.

   (a) Pick (and clearly define, perhaps with a picture) generalized coordinates for this problem.

   (b) Express the speed of the glider and the speed of the bob in terms of your coordinates.

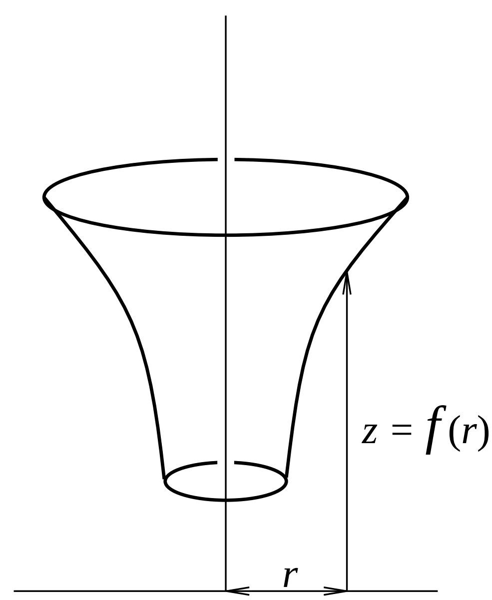
   (c) Express the potential energy of the glider and the potential energy of the bob in terms of your coordinates.

   (d) Express the Lagrangian in terms of your coordinates.

   (e) Find the Hamiltonian for this system.

   (f) Write down the differential equations for your coordinates/momenta found from the Hamiltonian.

3. Consider the problem of a object sliding frictionlessly down a cylindrically symmetric "drain" with height z a given function of the cylindrical radius r: $z = f(r)$. (The "drain" is a human sized object sitting in a convenience store on Earth.) Find the Lagrangian for this system. Find the Hamiltonian for this system. Is the Hamiltonian a constant of the motion? Is it equal to the energy? Notice that since θ (the polar angle in cylindrical coordinates) is cyclic there is a conserved quantity. Use this constant and the Hamiltonian to find an expression for time as an integral over r.



4. Consider the Lagrangian (m, q, and B are constants; $v_x = \dot{x}$, $v_y = \dot{y}$)

$$L_1 = \frac{1}{2}m(v_x^2 + v_y^2) + qBxv_y - V(x,y)$$

(a) Find the differential equations (Euler-Lagrange equations) describing the motion of the particle.

(b) Find the canonical momenta p_x and p_y

(c) Find the Hamiltonian.

(d) Find Hamilton's equations of motion from your Hamiltonian.

(e) In lecture I discussed the Lagrangian:

$$L_2 = \frac{1}{2}m(v_x^2 + v_y^2) - qByv_x - V(x,y)$$

and got exactly the same equations of motion but most everything else was different. Show that $L_1 - L_2 = \frac{d}{dt}$(something). FYI: I claim that these two facts are related.