18-38. The heat needed is found by integrating the heat capacity:

$$\begin{aligned}
Q &= \int_{T_i}^{T_f} cm\,dT = m\int_{T_i}^{T_f} c\,dT \\
&= (2)\int_{5.0^\circ\mathrm{C}}^{15.0^\circ\mathrm{C}} (0.20 + 0.14T + 0.023T^2)\,dT \\
&= (2.0)(0.20T + 0.070T^2 + 0.00767T^3)\Big|_{5.0}^{15.0}\ (\mathrm{cal}) \\
&= 81.8\ \mathrm{cal}\ .
\end{aligned}$$

18-39. (a) We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

- None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.
- The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.
- All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, we suppose that no ice melts. The temperature of the water decreases from $T_{Wi} = 25^\circ$C to some final temperature T_f and the temperature of the ice increases from $T_{Ii} = -15^\circ$C to T_f. If m_W is the mass of the water and c_W is its specific heat then the water (loses) heat

$$Q = c_W m_W\,(0 - T_{Wi}) - m_W L_F + c_I m_W\,(T_f - 0)$$

where L_F is the heat of fusion for water. If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = m_I c_I (T_f - T_{Ii})\ .$$

Since no energy is lost to the environment, these two heats must add to zero. Consequently,

$$c_W m_W\,(0 - T_{Wi}) - m_W L_F + c_I m_W\,(T_f - 0) + m_I c_I\,(T_f - T_{Ii}) = 0\ .$$

The solution for the equilibrium temperature is

$$\begin{aligned}
T_f &= \frac{m_W(c_W T_{Wi} + L_F) + m_I c_I T_{Ii}}{(m_W + m_I)c_I} \\
&= \frac{200\,\mathrm{g}\cdot[(4.19\,\mathrm{J/g{\cdot}K})\cdot 25^\circ\mathrm{C} + 333\,\mathrm{J/g}] + (100\,\mathrm{g})\cdot(2.22\,\mathrm{J/g{\cdot}K})\cdot(-15^\circ\mathrm{C})}{(200\,\mathrm{g} + 100\,\mathrm{g})(2.22\,\mathrm{J/g{\cdot}K})} \\
&= 126^\circ\mathrm{C}\ .
\end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at $T_f = 0^\circ$C, with mass m ($< m_I$) of the ice melted. The heat (lost, as $Q < 0$) by the water is

$$Q = m_W c_W (0 - T_{Wi})\ ,$$

and the heat absorbed by the ice is

$$Q = m_I c_I (0 - T_{Ii}) + m L_F\ ,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The heats add to zero, so

$$m_W c_W T_{Wi} = -m_I c_I T_{Ii} + m L_F\ .$$

This equation can be solved for the mass m of ice melted:

$$
\begin{aligned}
m &= \frac{m_W c_W T_{Wi} + m_I c_I T_{Ii}}{L_F} \\
&= \frac{(4.19\,\mathrm{J/g\cdot K})(200\,\mathrm{g})(25°\mathrm{C}) + (2.22\,\mathrm{J/g\cdot K})(100\,\mathrm{g})(-15°\mathrm{C})}{333\,\mathrm{J/g}} \\
&= 52.9\,\mathrm{g}\ .
\end{aligned}
$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

To be complete we now consider the third option. (Nonsense is expected.) The heat (lost, as $Q < 0$) by the water is

$$Q = m_W c_W (T_f - T_{Wi})\ ,$$

and the heat absorbed by the ice is

$$Q = m_I(c_I(0 - T_{Ii}) + L_F + c_W(T_f - 0))\,,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C, the second term is the energy required to melt all the ice, and the third is the energy required to bring the now melted ice up to the final temperature. The heats add to zero:

$$0 = m_W c_W (T_f - T_{Wi}) + m_I(c_I(-T_{Ii}) + L_F + c_W T_f)$$

So:

$$
\begin{aligned}
T_f &= \frac{m_W c_W T_{Wi} + m_I(c_I T_{Ii} - L_F)}{c_W(m_W + m_I)} \\
&= \frac{200\,\mathrm{g}\cdot(4.19\,\mathrm{J/g\cdot K})\cdot 25°\mathrm{C} + 100\,\mathrm{g}\cdot(2.22\,\mathrm{J/g\cdot K})\cdot(-15°\mathrm{C}) - 333\,\mathrm{J/g})}{(200\,\mathrm{g} + 100\,\mathrm{g})(4.19\,\mathrm{J/g\cdot K})} \\
&= -12.5°\mathrm{C}\ .
\end{aligned}
$$

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat (lost) by the water is

$$Q = m_W c_W (T_f - T_{Wi})$$

and the heat (absorbed) by the ice (and the water it becomes when it melts) is

$$Q = m_I c_I (0 - T_{Ii}) + m_I c_W (T_f - 0) + m_I L_F\ .$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats cancel,

$$0 = m_W c_W (T_f - T_{Wi}) + m_I c_I(-T_{Ii}) + m_I c_W T_f + m_I L_F\ .$$

The solution for T_f is

$$T_f = \frac{m_W c_W T_{Wi} + m_I c_I T_{Ii} - m_I L_F}{c_W(m_W + m_I)}\ .$$

Inserting the given values, we obtain $T_f = 2.52°\mathrm{C}$.

old exam: `2006/211t3_06.pdf #1`

$$m_{Cu} c_{Cu} (T_f - T_{Cu\,i}) + m_W c_W (T_f - T_{Wi}) + m_I c_I (0 - T_{Ii}) + m_I c_W (T_f - 0) + m_I L_F = 0$$

$$
\begin{aligned}
T_f(m_{Cu}c_{Cu} + m_W c_W + m_I c_W) &= m_{Cu}c_{Cu}T_{Cu\,i} + m_W c_W T_{Wi} + m_I c_I T_{Ii} - m_I L_F \\
T_f &= \frac{m_{Cu}c_{Cu}T_{Cu\,i} + m_W c_W T_{Wi} + m_I c_I T_{Ii} - m_I L_F}{m_{Cu}c_{Cu} + m_W c_W + m_I c_W} \\
&= \frac{250 \cdot 0.385 \cdot 22 + 250 \cdot 4.19 \cdot 95 + 50 \cdot 2.22 \cdot (-10) - 50 \cdot 333}{250 \cdot 0.385 + 250 \cdot 4.19 + 50 \cdot 4.19} \\
&= 61.8^\circ\mathrm{C}
\end{aligned}
$$

old exam: `2004/dean_211_thermo98.pdf #3`

$$
m_{Cu}c_{Cu}(T_f - T_{Cu\,i}) + m_W c_W (T_f - T_{Wi}) + m_I c_I (0 - T_{Ii}) + m_I c_W (T_f - 0) + m_I L_F = 0
$$

$$
\begin{aligned}
T_f(m_{Cu}c_{Cu} + m_W c_W + m_I c_W) &= m_{Cu}c_{Cu}T_{Cu\,i} + m_W c_W T_{Wi} + m_I c_I T_{Ii} - m_I L_F \\
T_f &= \frac{m_{Cu}c_{Cu}T_{Cu\,i} + m_W c_W T_{Wi} + m_I c_I T_{Ii} - m_I L_F}{m_{Cu}c_{Cu} + m_W c_W + m_I c_W} \\
&= \frac{200 \cdot 0.386 \cdot 20 + 150 \cdot 4.19 \cdot 75 + 20 \cdot 2.22 \cdot (-10) - 20 \cdot 333}{200 \cdot 0.386 + 150 \cdot 4.19 + 20 \cdot 4.19} \\
&= 52.7^\circ\mathrm{C}
\end{aligned}
$$

T1S.8 (Moore)

$$
\begin{aligned}
m_A c_A (T_f - T_A) + m_B c_B (T_f - T_B) &= 0 \\
T_f(m_A c_A + m_B c_B) &= m_A c_A T_A + m_B c_B T_B \\
T_f(u+1) &= u T_A + T_B \\
T_f(u+1) &= u T_A + (1 + u - u) T_B \\
T_f &= \frac{u}{1+u}\,(T_A - T_B) + T_B
\end{aligned}
$$

Clearly $m_A c_A \gg m_B c_B \Rightarrow u \to \infty \Rightarrow u/(1+u) \to 1 \Rightarrow T_f \to T_A$

On the other hand: $m_A c_A \ll m_B c_B \Rightarrow u \to 0 \Rightarrow u/(1+u) \to 0 \Rightarrow T_f \to T_B$

18-48. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero and the heat absorbed by the gas is equal to the work done by the gas: $Q = W$. In terms of the contributions of the individual parts of the cycle $Q_{AB} + Q_{BC} + Q_{CA} = W$ and $Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0\,\mathrm{J} - 20.0\,\mathrm{J} - 0 = -5.0\,\mathrm{J}$. This means 5.0 J of energy leaves the gas in the form of heat.

18-49. (a) The change in internal energy ΔE_{int} is the same for path iaf and path ibf. According to the first law of thermodynamics, $\Delta E_{\mathrm{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along iaf $\Delta E_{\mathrm{int}} = Q - W = 50\,\mathrm{cal} - 20\,\mathrm{cal} = 30\,\mathrm{cal}$. Along ibf $W = Q - \Delta E_{\mathrm{int}} = 36\,\mathrm{cal} - 30\,\mathrm{cal} = 6\,\mathrm{cal}$.

(b) Since the curved path is traversed from f to i the change in internal energy is $-30\,\mathrm{cal}$ and $Q = \Delta E_{\mathrm{int}} + W = -30\,\mathrm{cal} - 13\,\mathrm{cal} = -43\,\mathrm{cal}$.

(c) Let $\Delta E_{\mathrm{int}} = E_{\mathrm{int},\,f} - E_{\mathrm{int},\,i}$. Then, $E_{\mathrm{int},\,f} = \Delta E_{\mathrm{int}} + E_{\mathrm{int},\,i} = 30\,\mathrm{cal} + 10\,\mathrm{cal} = 40\,\mathrm{cal}$.

(d) The work W_{bf} for the path bf is zero, so —using the result (a)— $W_{ib} = 6\,\mathrm{cal}$. $Q_{ib} = \Delta E_{\mathrm{int}} + W = 12 + 6 = 18\,\mathrm{cal}$

(e) The work W_{bf} for the path bf is zero, so $Q_{bf} = E_{\mathrm{int},\,f} - E_{\mathrm{int},\,b} = 40\,\mathrm{cal} - 22\,\mathrm{cal} = 18\,\mathrm{cal}$.

18-72. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate $\pm$ sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in pV diagrams:

$$
\text{for straight line} \quad \text{Work} = \frac{p_i + p_f}{2}\,\Delta V
$$

The cycle represented by the "triangle" BC consists of three processes:

- "tilted" straight line from $(1.0\,\mathrm{m}^3, 40\,\mathrm{Pa})$ to $(4.0\,\mathrm{m}^3, 10\,\mathrm{Pa})$, with

$$\text{Work} = \frac{40\,\mathrm{Pa} + 10\,\mathrm{Pa}}{2}\left(4.0\,\mathrm{m}^3 - 1.0\,\mathrm{m}^3\right) = 75 \text{ J}$$

- horizontal line from $(4.0\,\mathrm{m}^3, 10\,\mathrm{Pa})$ to $(1.0\,\mathrm{m}^3, 10\,\mathrm{Pa})$, with

$$\text{Work} = (10\,\mathrm{Pa})\left(1.0\,\mathrm{m}^3 - 4.0\,\mathrm{m}^3\right) = -30 \text{ J}$$

- vertical line from $(1.0\,\mathrm{m}^3, 10\,\mathrm{Pa})$ to $(1.0\,\mathrm{m}^3, 40\,\mathrm{Pa})$, with

$$\text{Work} = \frac{10\,\mathrm{Pa} + 40\,\mathrm{Pa}}{2}\left(1.0\,\mathrm{m}^3 - 1.0\,\mathrm{m}^3\right) = 0$$

Thus, the total work during the BC cycle is $75 - 30 = 45$ J. During the BA cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with $\text{Work} = (40\,\mathrm{Pa})(-3.0\,\mathrm{m}^3) = -120$ J. Therefore, the total work during the BA cycle is $75-120 = -45$ J. Note that the area of either triangle is: $\frac{1}{2}(30\,\mathrm{Pa} \times 3\,\mathrm{m}^3) = 45$ J, so we can directly find the net work if we apply the proper sign.