16-6. (a) The amplitude is $y_m = 6.0\,\text{cm}$.

(b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 100\,\text{cm}$.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0\,\text{Hz}$.

(d) The wavespeed is $v = \lambda f = (100\,\text{cm})(2.0\,\text{Hz}) = 200\,\text{cm/s}$.

(e) The wave propagates in the negative x direction, since the argument of the trig function is $kx+\omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\max} = 2\pi f y_m = \left(4.0\pi\,\text{s}^{-1}\right)(6.0\,\text{cm}) = 75.4\ \text{cm/s}\ .$$

(g) $y(3.5\,\text{cm}, 0.26\,\text{s}) = (6.0\,\text{cm})\sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.03\,\text{cm}$.

16-13. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope: $\mu = m/L = (0.0600\,\text{kg})/(2.00\,\text{m}) = 0.0300\,\text{kg/m}$. Thus

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\ \text{m/s}\ .$$

16-19. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15\,\text{cm}$ and again with the same slope at about $x = 55\,\text{cm}$, so $\lambda = 55\,\text{cm} - 15\,\text{cm} = 40\,\text{cm} = 0.40\,\text{m}$.

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25\times10^{-3}\,\text{kg/m}}} = 12\,\text{m/s}\ .$$

(d) The frequency is $f = v/\lambda = (12\,\text{m/s})/(0.40\,\text{m}) = 30\,\text{Hz}$ and the period is $T = 1/f = 1/(30\,\text{Hz}) = 0.0333\,\text{s}$.

(e) The maximum string speed is $u_m = \omega y_m = 2\pi f y_m = 2\pi(30\,\text{Hz})(5.0\,\text{cm}) = 942\,\text{cm/s} = 9.42\,\text{m/s}$.

(f) The string displacement is assumed to have the form $y(x,t) = y_m \sin(kx + \omega t + \phi)$. The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40\,\text{m}) = 15.7\,\text{m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30\,\text{Hz}) = 188\,\text{rad/s}$,

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is $4.0\times10^{-2}\,\text{m}$. The formula for the displacement gives $y(0,0) = y_m \sin\phi$. We wish to select ϕ so that $5.0\times10^{-2}\sin\phi = 4.0\times10^{-2}$. The solution is either 0.927 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.927\,\text{rad}$.

(i) A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. The amplitude is $y_m = 5.0\times10^{-2}\,\text{m}$
The expression for the displacement is

$$y(x,t) = (5.0\times10^{-2}\,\text{m})\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right]\ .$$

16-37. Adding the complex amplitudes: $4.6e^{0i} + 5.6e^{.8\pi i} = (.007, 3.29) = 3.29\angle 1.55^r$
Now:

$$\sum_{k=1}^{N} a_k \sin(\omega t + \delta_k) = \text{Im}\left[\left(\sum_{k=1}^{N} a_k e^{i\delta_k}\right)\ e^{i\omega t}\right] = A\sin(\omega t + \phi)$$

where: $\displaystyle\sum_{k=1}^{N} a_k e^{i\delta_k} = A\ e^{i\phi}$, so here:

$$\sum_{k=1}^{N} a_k \sin(\omega t + \delta_k) = 3.29\sin(\omega t + 1.55)$$

(a) 3.29 mm

(b) 1.55^r

(c) The sum of the first two waves is the "vector" $3.29\angle 1.55^r$; with vector addition maximum extension is achieved if the vectors added together are in the same direction. Hence to maximize the amplitude of this wave the phase of the third wave should be the same as the sum of the first two: 1.55^r

16-43. (a) Eq. 16-26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\mathrm{N}}{7.2 \times 10^{-3}\,\mathrm{kg/m}}} = 144 \text{ m/s} .$$

(b) From the Figure, we find the wavelength of the standing wave to be $\lambda = (2/3)(90\,\mathrm{cm}) = 60\,\mathrm{cm}$.

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.4 \times 10^2\,\mathrm{m/s}}{0.60\,\mathrm{m}} = 241 \text{ Hz} .$$

16-50. (a) The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Longrightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \ldots$$

so that the values of x lying in the allowed range are $x = 0$, $x = 0.20$ m, and $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16-56 through Eq. 16-60, we obtain

$$y_1 = 0.020 \sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020 \sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -\,(0.040)\,(40\pi) \sin(5\pi x) \sin(40\pi t)$$

which vanishes (for all x) at times such $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Longrightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \ldots$$

so that the values of t lying in the allowed range are $t = 0$, $t = 0.025$ s, and $t = 0.050$ s.

16-82. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and $L = 2v/f = 2(400\,\mathrm{m/s})/(600\,\mathrm{Hz}) = 1.33\,\mathrm{m}$.

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx)\cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is $k = 2\pi/\lambda = 2\pi f/v = 2\pi(600\,\mathrm{Hz})/(400\,\mathrm{m/s}) = 9.42\,\mathrm{m}^{-1}$ and the angular frequency is $\omega = 2\pi f = 2\pi(600\,\mathrm{Hz}) = 3770\,\mathrm{rad/s}$. y_m is 2.0 mm. The displacement is given by

$$y(x,t) = (2.0\,\mathrm{mm}) \sin[(9.42\,\mathrm{m}^{-1})x] \cos\left[(3770\,\mathrm{s}^{-1})t\right] .$$

16-83. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed'). Thus, $\lambda = 2(0.90\,\mathrm{m})/4 = 0.45\,\mathrm{m}$. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is $\mu = m/L = (0.044\,\mathrm{kg})/(0.90\,\mathrm{m}) = 0.0489$ kg/m. Thus, using Eq. 16-26, we obtain the tension: $\tau = v^2\mu = (27)^2(0.0489) = 35.6$ N.