Physics 211 EXAM 3 2006 December 11

Answer 5 of the following 6 questions

Physical Constants

$\sigma = 5.6705 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$
$R = 8.3145\ \mathrm{J/(K \cdot mol)}$
$N_A = 6.0221 \times 10^{23}$
$k_B = 1.3807 \times 10^{-23}\ \mathrm{J/K}$
$1\ \mathrm{eV} = 1.6022 \times 10^{-19}\ \mathrm{J}$
$1\ \mathrm{atm} = 1.0133 \times 10^{5}\ \mathrm{Pa}$

Properties of H_2O

$L_V = 2256\ \mathrm{J/g}$
$c_w = 4.19\ \mathrm{J/(g \cdot K)}$
$L_f = 333\ \mathrm{J/g}$
$c_i = 2.22\ \mathrm{J/(g \cdot K)}$
$\rho_w = 1000\ \mathrm{kg/m^3}$

1. A 250 g copper container is at a temperature of 22°C. Water (250 g at 95°C) and ice (50 g at −10°C) are placed in the container. What will be the equilibrium temperature of this system? (specific heat of copper: $c_{\mathrm{Cu}} = 0.385\ \mathrm{J/(g \cdot K)}$)

2. The entropy of 1 g of liquid water (at $T = 25$°C and $p = 100$ kPa) is 3.883 J/K.

   (a) How many microstates are consistent with the above macro variables?

   (b) A small amount of heat dQ is added to the water and the number of microstates increases to a billion (10^9) times the previous number. Find the change in entropy, dS; find the amount of heat added, dQ.

   (c) With that added heat, the temperature of the water will change. Find the change in temperature, dT.

   Rhetorical question: do you think these changes are measurable?

3. A selection of rows from a run of `StatMech` with $N_A = 100$ (atoms), $N_B = 100$, and total energy $U = 2000\varepsilon$ can be found as the final page of this exam. This Einstein solid has $\varepsilon = .005$ eV.

   (a) What is the dimensionless entropy of system A (i.e., S_A/k_B) for $U_A = 0\varepsilon$? $U_A = 10\varepsilon$? $U_A = 100\varepsilon$? $U_A = 1000\varepsilon$?

   (b) Approximate $\dfrac{\partial S}{\partial U}$ as a (small) finite difference $\dfrac{\Delta S}{\Delta U}$. Use this result to find an equation for the temperature of this Einstein solid. Simplify your result using the properties of logarithms. Calculate T_A (the temperature of system A) using $\Delta U = 1\varepsilon$ for $U_A = 100\varepsilon$, $U_A = 200\varepsilon$, $U_A = 1900\varepsilon$, and $U_A = 2000\varepsilon$.

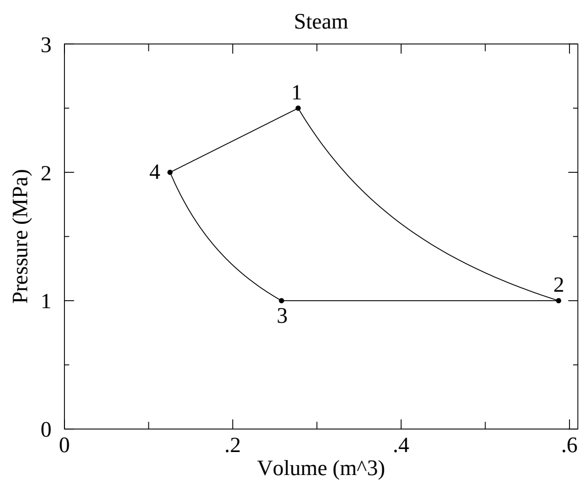
   (c) Compare the change in temperature that results when 100ε of heat is added in the process: $U_A : 100\varepsilon \to 200\varepsilon$ to the change in temperature that results in the process: $U_A : 1900\varepsilon \to 2000\varepsilon$. Does the specific heat seem approximately constant? Which case has the smaller specific heat?

4. The following problem is based on "steam tables"—tables of V, T, E_{int}, S etc. which substitute for the nice equations like $pV = nRT$, $\Delta S = nC_p \ln(T_f/T_i)$ etc. that apply only to the mythical ideal gas. Again steam is a non-ideal gas; you must use the tabulated V, T, E_{int}, S etc. not formulas based on $pV = nRT$.

   Starting at a pressure of 2.5 MPa, volume 0.2777 m^3, and temperature 1232°C, 1 kg of water vapor goes through the following cycle:

   - The steam expands adiabatically to pressure of 1 MPa.
   - In an isobaric process, the steam is cooled until the volume is 0.2579 m^3.
   - In an isothermal process (at 300°C) the steam is compressed until a pressure of 2 MPa is achieved.
   - A straightline process returns to the initial state.

   The following graph displays this cycle and table reports state variables at the labelled points.



point	Volume (m^3)	Pressure (MPa)	Temperature (°C)	E_{int} (kJ)	Entropy (kJ/K)
1	0.2777	2.5	1232	4531	8.912
2	0.5871	1.0	1000	4051	8.912
3	0.2579	1.0	300	2793	7.123
4	0.1255	2.0	300	2773	6.766

   (a) Estimate the value of the ratio of specific heats γ based on the adiabatic expansion $1 \to 2$. (In fact this ratio is not constant in this non-ideal gas, so it is of no use in the following parts.)

   (b) Use the first law of thermodynamics to find the heat removed in the isobaric compression $2 \to 3$.

   (c) Use ΔS to find the heat removed in the isothermal compression $3 \to 4$

   (d) How much heat was added in the straightline expansion $4 \to 1$?

   (e) Use all your heats to find the *net work* performed by this cycle.

5. Consider the following cycle starting with 0.5 m^3 of a monoatomic ideal gas at a pressure of 2 atm and a temperature of 500 K

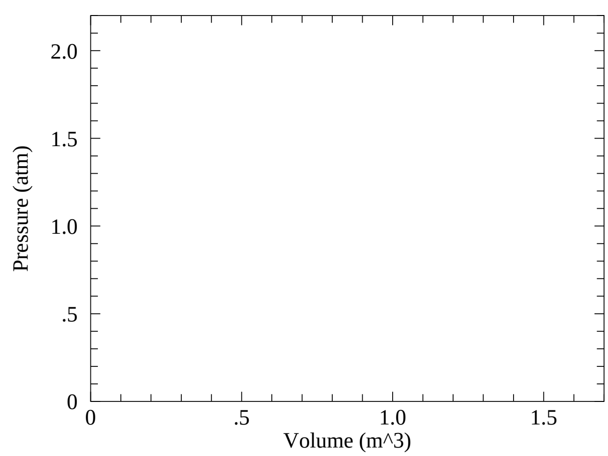
   (a) In a constant-temperature (a.k.a., isothermal) process, the volume is expanded to 1 m^3.

   (b) In an adiabatic process the volume is expanded until a pressure of 0.5 atm is obtained.

   (c) In a constant-pressure (a.k.a., isobaric) process, the volume is returned to 0.5 m^3.

   (d) A constant-volume (a.k.a., isochoric) process returns the temperature to 500 K.

   On the below graph, plot each leg of this cycle. This will require calculating various pVT values at the end of each cycle. Fill in the below table giving the sign $(+, -, 0)$ of the change in internal energy and entropy for each leg of the cycle. Report the amount of 'working fluid' (i.e., the gas) in moles.



path	ΔE_{int}	ΔS
a		
b		
c		
d		

6. Carbon dioxide (CO_2, that's ^{12}C and ^{16}O) is a linear molecule. Report (source and value) the contributions you expect to the number of degrees of freedom f at room temperature. Report the per-mole and per-gram constant-pressure specific heats (c_p) you expect for this f. CO_2's lowest vibrationally excited states are at 0.083 eV and 2×0.083 eV. Approximate the sum over all states by just these two excited states and the ground state. Find the partition function and the probability CO_2 is in the 0.083 eV state at 300 K.